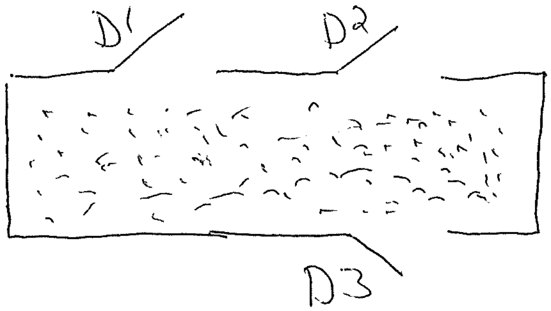
10/22/19

M159: Single-life (one person); Multiple Decrements



D - decrement (door)

Notation:

T_x = rvr time until the departure of (x)

J = rvr mode (type) of departure

$J = \{1, 2, 3\}$

$J = 1 \iff (x)$ departs by D1

${}_nq_x^{(j)} = Pr((x)$ departs within n years by decrement $j)$

$\cancel{{}_np_x^{(j)}}$ never used, since there's no good way of defining the event

tau "total"

${}_nq_x^{(\tau)} = Pr((x)$ departs within n years$)$

↳ "implied" for any reason

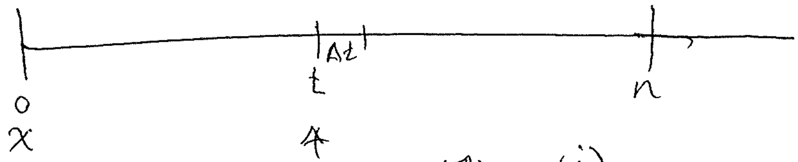
${}_np_x^{(\tau)} = Pr((x)$ survives for n years$)$

↳ "implied" all decrements

Remarks: 1) ${}_np_x^{(\tau)} + {}_nq_x^{(\tau)} = 1$

2) ${}_nq_x^{(\tau)} = {}_nq_x^{(1)} + {}_nq_x^{(2)} + \cdots = \sum_j {}_nq_x^{(j)}$

3) ${}_nq_x^{(j)}$:



$$Pr = {}_tp_x^{(\tau)} \cdot \mu_{x+t}^{(j)} \cdot \Delta t$$

$$\therefore \; {}_nq_x^{(j)} = \int_0^n {}_tp_x^{(\tau)} \cdot \mu_{x+t}^{(j)} \, dt$$

Likewise $\; {}_nq_x^{(\tau)} = \int_0^n {}_tp_x^{(\tau)} \cdot \mu_{x+t}^{(\tau)} \, dt$

$$\mu_{x+t}^{(\tau)} = \mu_{x+t}^{(1)} + \mu_{x+t}^{(2)} + \cdots$$

Test 8 Review:

MIS7 (Multiple Lives)

$$\mathring{e}_{\sigma} = \int_0^{\infty} {}_tp_{\sigma}\, dt \qquad e_{\sigma} = \sum_k {}_kp_{\sigma}$$

$\sigma = x{:}\overline{n}|$

$$\mathring{e}_{x:\overline{n}|} = \int_0^{n} {}_tp_x\, dt \qquad e_{x:\overline{n}|} = \sum_{k=1}^{n} {}_kp_x$$

Recursion:

$$\mathring{e}_{x:\overline{n}|} \overset{k<n}{=\!=} \mathring{e}_{x:\overline{k}|} + {}_kp_x \cdot \mathring{e}_{x+k:\overline{n-k}|}$$

MIS8 (Independent Lives)

$${}_np_{xy} = {}_np_x \cdot {}_np_y$$

$${}_nq_{\overline{xy}} = {}_nq_x \cdot {}_nq_y$$

$$\mu_{xy}(t) = \mu_x(t) + \mu_y(t)$$

Contingent Probabilities $({}_nq^1_{xy}, \ldots)$

Examples: (MIS8 Exercises)

1) & 2) male mortality is DML ($\omega = 120$)
female mortality is CF ($\mu = \frac{1}{40} = .025$)

$${}_tp^m_x = \frac{120-x-t}{120-x} \qquad {}_tp^f_x = e^{-.025t}$$

L-TAM Module 1 Section 8 Exercises

Unless told or implied otherwise, assume all lives are independent.

For numbers 1 and 2, suppose deaths for males is uniformly distributed over the interval [0, 120] and female mortality follows an exponential distribution with mean 40.

1. Determine ${}_{30}q_{40:45}$ where (40) is female and (45) is male.

2. If (40) is male and (45) is female, determine

   (a) ${}_{10|30}q_{\overline{40:45}}$

   (b) ${}_{10}p_{\overline{40:45}}$

   (c) ${}_{30}q_{\overline{50:55}}$ where (50) is male and (55) is female

   (d) Is ${}_{10|30}q_{\overline{40:45}} = {}_{10}p_{\overline{40:45}} \cdot {}_{30}q_{\overline{50:55}}$? (Comparing (a), (b), and (c).)

   (e) ${}_{t}p_{40:45}$ for $t < 80$

   (f) ${}_{t}p_{40:45}$ for $t > 80$

   (g) $\overset{o}{e}_{40:45}$

   (h) $\overset{o}{e}_{\overline{40:45}}$

   (i) ${}_{20}q_{40:45}$

   (j) ${}_{20}q_{\overset{1}{40}:45}$

   (k) ${}_{20}q_{40:\overset{1}{45}}$

   (l) ${}_{10}q_{\overline{40:45}}$

   (m) ${}_{10}q_{\overset{2}{40}:45}$

   (n) ${}_{10}q_{40:\overset{2}{45}}$

   (o) ${}_{30}q_{\overset{1}{40}:45} - {}_{30}q_{40:\overset{2}{45}}$

1) $${}_{30}q_{40:45} = 1 - {}_{30}p_{40:45}$$

$${}_{30}p_{40:45} = {}_{30}p^{f}_{40} \cdot {}_{30}p^{m}_{45}$$

2a) $${}_{10|30}q_{\overline{40:45}} = {}_{40}q_{\overline{40:45}} - {}_{10}q_{\overline{40:45}}$$

$$= {}_{40}q^{m}_{40} \cdot {}_{40}q^{f}_{45} - {}_{10}q^{m}_{40} \cdot {}_{10}q^{f}_{45}$$

Remark:

$${}_{10|30}q_{\overline{40:45}} \neq {}_{10}p_{\overline{40:45}} \cdot {}_{30}q_{\bigcirc}$$

↳ we're not guaranteed to have (50) & (55)

2(h) $$\mathring{e}_{\overline{40:45}} = \underbrace{\mathring{e}^{m}_{40}}_{=40} + \underbrace{\mathring{e}^{f}_{45}}_{=40} - \underbrace{\mathring{e}_{40:45}}_{2(g)}$$